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PRELIMINARY REVIEW OF STATISTICAL TREATMENT OF S-GLASS/EPOXY STRESS RUPTURE DATA

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The purpose of this report is to provide statistically derived tabular design data to predict the estimated stress rupture life of unidirectional E-glass/epoxy composites subjected to sustained tensile loading. The maximum likelihood estimated life (MLEL) is tabulated for various stress levels of the fiber and quantile level. The lower bound lifetime with confidence intervals of 95 and 99 percent are also tabulated for the various fiber stress and quantile levels. The estimated lifetimes were calculated from accelerated stress rupture tests of T. T. Chiao (Lawrence Livermore Laboratory) who used uniaxial strand samples; therefore, the estimated lifetimes reported are considered to be conservative.

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PREFACE

This report covers work performed under Air Force Contract No. F33615-82-C-5039. The work was administered under the direction of the Materials Laboratories, Air Force Systems Command, Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio. Mr. Bennie Cohen (AFWAL/MLSA) was the Program Project Engineer.

The Principal Investigator on this program was Dr. James A. Snide, Director of Graduate Materials Engineering at the University of Dayton.

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I. INTRODUCTION

In May 1981, an aircraft sustained major damage when a high-pressure nitrogen storage bottle installed in the aircraft ruptured without warning. The bottle was a filament-wound S-glass/epoxy composite structure containing nitrogen at approximately 5000 psi pressure. The pressurized bottle is used to inflate the aircraft crew module impact attenuation bag and the pressure in the bottle is not released during the service life of the bottle unless the attenuation bag is employed. The bottle that failed had been in service and, therefore, pressurized for eight years when the failure occurred. The Accident Investigation Board at Sacramento Air Logistic Center concluded that the failure was due to a stress rupture failure of the nitrogen bottle.

The stress rupture of composites under sustained tensile loading has been extensively studied by T. T. Chiao and associates (References 1-2) at the Lawrence Livermore National Laboratories during the past decade. This work has shown that the stress rupture life decreased as the stress developed in the fibers is increased relative to the ultimate strength of the fibers. For example, the stress rupture test of one hundred S-glass/epoxy composites loaded to 83.5% of the ultimate strength of the S-glass fiber failed in the time range of 0.01 hours to 8.55 hours. A similar group of strand samples loaded to 65% of the fiber strength exhibited initial failure at 8.6 hours and all the samples had failed at 5014 hours. The fibers in the failed bottles were loaded to 36-40% of their ultimate flow strength; therefore, there was a 3-4% probability of stress rupture failure of the bottle after eight years of service. As a result of this investigation, all the remaining bottles of similar service were replaced and the old bottles were destroyed.

The Sacramento Air Logistic Center recommended that the appropriate AFSC design handbooks should be updated to provide information concerning the stress rupture failure of composites subject to sustained tensile loading. This report is an initial attempt to summarize the experimental stress rupture data on S-glass/epoxy of T. T. Chiao into tabular form which can then be incorporated into the appropriate AFSC design manuals. In addition, the methodology of the statistical analysis of strand data to calculate the Maximum Likelihood Estimate Life (MLEL) and lower bounds for various fiber stress levels is discussed.

II. BACKGROUND

In the report (Reference 1) by T. T. Chiao, R. E. Glaser and R. L. Moore, approximately 10 years of stress-rupture data for impregnated fiber strands were statistically analyzed. The data were collected at seven stress levels ranging from 480 ksi to 189 ksi representing values from 84% to 33% of strand failure stress. The S-glass/epoxy strand data that are checked in the above report are listed in Table 1.

Any statistical treatment of the experimental data must consider the following difficulties:

1. The data were collected at seven different stress levels (σ) and the parameters (α and β) of the statistical model for the strand lifetime are dependent on the stress.
2. The nature of the relationship between σ and the parameters (α and β) of the strand lifetime distribution must be determined from the data.
3. There is little data for values of σ below 50% UTS.

The probability distribution of the time to failure for a strand at a given stress level is determined if α and β are known. The usual method of analysis is to estimate α and β separately for each stress level. The difficulty of using this approach for the S-glass data is that very little

TABLE 1

SUMMARY OF STRESS RUPTURE DATA
FOR S-GLASS/EPOXY STRANDS

STRESS, KSI	PERCENTILE OF FAILURE STRESS (% UTS)	# STRANDS TESTED	# EXACT FAILURE TIMES	# GROUPED FAILURE TIMES	#CENSORED TIMES
479.5	(84)	117	114	3	0
426.3	(75)	104	104	0	0
373.0	(65)	115	105	0	10
332.0	(58)	98	21	13	64
286.0	(50)	46	4	3	39
228.8	(40)	100	2	0	98
188.7	(33)	100	1	0	99

data are available for stress levels below 286 ksi, and no useful statistical estimates can be constructed using only the data for the stress levels of interest below 286 ksi.

What is new in the statistical treatment of this data is that a model is used that incorporates the dependence of the parameters α and β on the stress level in the Weibull time to failure distribution. Instead of estimating the parameters α and β separately at each stress level, new parameters are introduced that model the dependence of α and β on the applied stress level. These new parameters are then estimated from all the data using the method of maximum likelihood. Once the parameters of the model relating the dependence of α and β on the stress level have been estimated from the data, estimates of quantile points in the failure time distribution can be determined.

III. ANALYSIS

The resulting lifetimes of the stress-rupture tests at each given stress level were modeled as a Weibull distribution. The distribution function for a Weibull distribution is

$$F(t) = 1 - \exp[-(t/\beta)^{\alpha}] \qquad (1)$$

where $F(t)$ is the probability that the lifetime of a given strand subject to a stress (σ) will be less than or equal to t, and α and β are the shape and scale parameters of the Weibull model to be estimated from the experimental data. These parameters, α and β of the Weibull model, vary with the given stress level (σ).

An insight into the meaning of the parameters α and β in the Weibull distribution of the S-glass data can be obtained by noting that the mean lifetime under a Weibull model is $\beta\Gamma(1 + \frac{1}{\alpha})$, where $\Gamma(1 + \frac{1}{\alpha})$ can be found from tables of the gamma function (Reference 3). For the S-glass data, α was approximately one for all stress levels. The mean life of a strand is therefore approximately $\beta\Gamma(2) = \beta$.

The reliability function R(t), which is the probability that the lifetime of a strand subject to a stress (σ) will exceed t, is related to the distribution function as follows:

$$R(t) = 1 - F(t) = \exp\left[-\left(\frac{t}{\beta}\right)^{\alpha}\right] \tag{2}$$

Once the parameters α and β are estimated, the reliability can be determined for any stress level σ.

To illustrate the effect of the α parameter on the shape of the Weibull lifetime distribution, Figure 1 below shows how the shape of the Weibull density changes from an exponential type curve when α is 1 to an almost bell-shaped curve for $\alpha = 4$.

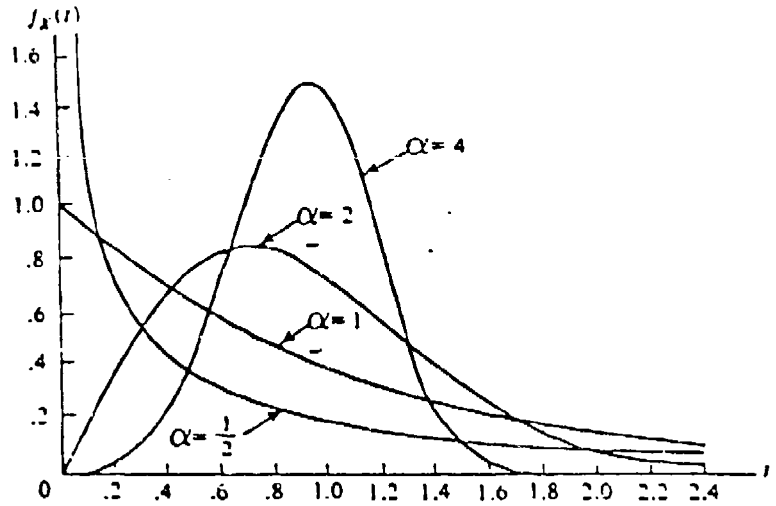

Figure 1. Graphs of the Weibull probability density functions (β = 1).

The basic method of analysis used in this paper to estimate the values of α and β of the Weibull model is known as the method of maximum likelihood. In this method, the joint distribution of the observed data (called the likelihood function) is regarded as a function of the parameters α and β. The values $\hat{\alpha}$ and $\hat{\beta}$ that maximized the likelihood function are referred to as maximum likelihood estimates of the parameters α and β.

These values can then be substituted into the reliability equations to create the tables.

As stated earlier, the parameters α and β vary with each stress level. The following two relationships were used to describe how α and β varied with the stress level for the data being analyzed.

$$\frac{1}{\alpha(\sigma)} = \theta_1 + \theta_2\sigma, \tag{3}$$

$$\ln(\beta(\sigma)) = \theta_3 + \theta_4\sigma + \theta_5\sigma^2 + \theta_6\sigma^3 \tag{4}$$

θ_n are constant coefficients that are to be determined. Other representations of the relationship between α and β and the stress level (σ) were considered but the linear model for α and cubic model for β fit best with the experimental data (Reference 4).

Furthermore, to insure that $\beta(\sigma)$ is a decreasing function of σ, the constraint $\theta_4 = -300(\theta_5 + 225\theta_6)$ was imposed. This constraint is obtained by taking the derivative of Eq. 4 and setting it equal to zero when $\sigma = 150$, making $\beta(\sigma)$ a decreasing function of σ for values of σ from 500 ksi to 150 ksi. This constraint is derived as follows,

$$b(\sigma) = \ln[\beta(\sigma)]$$

$$b(\sigma) = \theta_3 + \theta_4\sigma + \theta_5\sigma^2 + \theta_6\sigma^3$$

and taking the derivative

$$b'(\sigma) = \theta_4 + 2\theta_5\sigma + 3\theta_6\sigma^2$$

and setting it equal to zero while $\sigma = 150$,θ_4 is obtained.

$$\theta_4 = -300(\theta_5 + 225\theta_6) \tag{5}$$

Using these representations of $\alpha(\sigma)$ and $\beta(\sigma)$, Eqs. 3 and 4 were then substituted into the likelihood equation which was then maximized as a function of the six variables (θ_1, θ_2, θ_3, θ_4, θ_5, θ_6). The maximization was subject to the constraint of Eq. 5 (Reference 3).

The maximum likelihood estimates $(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5, \hat{\theta}_6)$ are then obtained by setting five partial derivatives of the likelihood equations equal to zero and solving the resulting five equations and five unknowns. (There are only five unknowns since θ_4 is a function of θ_5 and θ_6.)

This system of equations that must be solved is nonlinear and presents a difficult numerical analysis problem. R. E. Glaser, developed a Fortran program to solve this system of equations. However, his method is highly dependent upon close initial estimates of $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$. If good initial estimates are not available, the program may not converge to the maximum vector quantities. To obtain good initial estimates of the six unknown parameters the following technique is used. Estimates of α and β are obtained for each different stress level by the maximum likelihood method. Then first and third degree polynomials are fit to the $\sigma - \alpha$ and $\sigma - \beta$ pairs, respectively, (values of $\hat{\alpha}$ and $\hat{\beta}$ obtained at each stress level) by the method of least squares. The least square estimations of the coefficients in the linear and cubic models are then used as initial estimates of the unknown parameters $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$ in the Fortran program developed by R. E. Glaser.

Once the parameters α and β of the Weibull model have been estimated by the above procedure, quantile points in the lifetime distribution of the S-Glass/Epoxy composite can be estimated. A quantile is a point in the distribution that has a given percentage of the distribution less than it. For example, the 0.1 quantile in a lifetime distribution is a time such that the probability of a strand failing before that time is 0.1. Using the maximum likelihood estimates ($\hat{\alpha}$ and $\hat{\beta}$), the pth quantile in the lifetime distribution is obtained by setting

$$p = 1 - \exp\left[-\left(\frac{t}{\hat{\beta}}\right)^{\hat{\alpha}}\right] \quad (6)$$

and solving for t. The solution, yielding the pth quantile is:

$$t = \exp\left[\frac{1}{\hat{\alpha}} \ln(-\ln(1 - p)) + \ln(\hat{\beta})\right] \quad (7)$$

IV. RESULTS

Table 2 was generated from Eq. 7 and gives the maximum likelihood estimated life (MLEL) for failure probabilities ranging from 0.0001 to 0.1 and for stress levels ranging from 26% to 45% of the ultimate flow stress (UTS).

For example, at a stress level of 30% (UTS) and a quantile value of .001, the table value is 20.49 years. This means that the maximum likelihood estimate of the point in the distribution of lifetimes under which there is a probability of failure of 0.001 is 20.49 years. This estimate has uncertainty in it since it is based on estimates of α and β obtained from the experimental data. Tables 3 and 4 give lower confidence bounds at 95% and 99% levels of confidence, respectively, for quantiles in the lifetime distribution under which the probability of a strand failing is either 0.0001, 0.001, 0.01, or 0.1. The second set of tables are more conservative and reflect the size of possible errors in the estimation procedures. At the 95% level of confidence, the values in the table were constructed so that there is a probability of 0.95 that the true quantile points in the lifetime distribution are not lower than the table values. To illustrate further the use of the tables, suppose that an estimate of the 0.001 quantile point in the lifetime distribution of a strand subject to a stress of 30% (UTS) is desired. This is a point in the distribution below which there is a probability of 0.001 that a strand will fail. Using Table 2 for the MLEL, the estimate is found to be 20.49 years. This is the maximum likelihood estimate of the 0.001 quantile of this

TABLE 2

ESITMATED LIFETIME OF S-GLASS/EPOXY COMPOSITE
UNDER SUSTAINED TENSILE LOADING

MLEL CALCULATIONS (YEARS)
(MAXIMUM LIKELIHOOD ESTIMATED LIFE)

		QUANTILE LEVEL			
% Fiber Stress	Stress (KSI)	0.0001 MLEL (YRS)	0.001 MLEL (YRS)	0.01 MLEL (YRS)	0.1 MLEL (YRS)
26	148.7	1.87	26.49	375.8	5600.8
27	154.4	1.82	25.86	368.9	5528.6
28	160.2	1.72	24.57	352.4	5310.1
29	165.9	1.58	22.73	327.8	4966.6
30	171.6	1.42	20.49	297.1	4527.0
31	177.3	1.24	18.02	262.7	4024.5
32	183.0	1.06	15.46	226.7	3492.3
33	188.8	0.88	12.97	191.1	2960.4
34	194.5	0.72	10.63	157.5	2453.5
35	200.2	0.57	8.52	127.0	1989.5
36	205.9	0.45	6.69	100.3	1579.7
37	211.6	0.34	5.15	77.6	1229.2
38	217.4	0.26	3.89	58.9	938.1
39	223.1	0.19	2.88	43.9	702.7
40	228.8	0.14	2.10	32.1	517.1
41	234.5	0.10	1.50	23.1	374.1
42	240.2	0.07	1.06	16.4	266.2
43	246.0	0.05	0.73	11.4	186.6
44	251.7	0.03	0.50	7.8	128.9
45	257.4	0.02	0.34	5.3	87.8

TABLE 3

ESTIMATED LIFETIME OF S-GLASS/EPOXY COMPOSITE
UNDER SUSTAINED TENSILE LOADING

LOWER BOUND CALCULATIONS (YEARS)
95% CONFIDENCE INTERVAL

		QUANTILE LEVEL			
% FIBER STRESS	STRESS (KSI)	0.0001 LB (YRS)	0.001 LB (YRS)	0.01 LB (YRS)	0.1 LB (YRS)
26	148.7	0.15	3.9	94.9	1991.0
27	154.4	0.15	3.9	95.0	1972.7
28	160.2	0.15	3.9	92.6	1906.4
29	165.9	0.14	3.7	88.0	1798.2
30	171.6	0.14	3.5	81.6	1656.7
31	177.3	0.13	3.2	73.9	1491.9
32	183.0	0.11	2.8	65.3	1314.2
33	188.8	0.10	2.4	56.5	1133.1
34	194.5	0.08	2.1	47.8	957.0
35	200.2	0.07	1.7	39.6	792.2
36	205.9	0.06	1.4	32.2	643.2
37	211.6	0.05	1.1	25.6	512.6
38	217.4	0.04	0.9	20.0	401.2
39	223.1	0.03	0.7	15.4	308.6
40	228.8	0.02	0.5	11.6	233.5
41	234.5	0.02	0.4	8.6	173.8
42	240.2	0.01	0.3	6.3	127.4
43	246.0	0.01	0.2	4.5	92.0
44	251.7	0.01	0.1	3.2	65.5
45	257.4	0.00*	0.1	2.2	46.1

*LESS THAN OR EQUAL TO .005

TACLE 4

ESTIMATED LIFETIME OF S-GLASS/EPOXY COMPOSITE
UNDER SUSTAINED TENSILE LOADING

LOWER BOUND CALCULATIONS (YEARS)
99% CONFIDENCE INTERVAL

		QUANTILE LEVEL			
% FIBER STRESS	STRESS (KSI)	0.0001 LB (YRS)	0.001 LB (YRS)	0.01 LB (YRS)	0.1 LB (YRS)
26	148.7	0.05	1.7	53.7	1297.5
27	154.4	0.05	1.8	54.2	1287.6
28	160.2	0.05	1.8	53.3	1247.5
29	165.9	0.05	1.7	51.1	1180.8
30	171.6	0.05	1.7	47.8	1092.7
31	177.3	0.05	1.5	43.7	989.3
32	183.0	0.04	1.4	39.0	876.9
33	188.8	0.04	1.2	34.1	761.4
34	194.5	0.03	1.1	29.2	648.1
35	200.2	0.03	0.9	24.4	541.1
36	205.9	0.02	0.7	20.1	443.4
37	211.6	0.02	0.6	16.2	356.9
38	217.4	0.02	0.5	12.8	282.3
39	223.1	0.01	0.4	10.0	219.5
40	228.8	0.01	0.3	7.6	168.0
41	234.5	0.01	0.2	5.7	126.5
42	240.2	0.01	0.2	4.2	93.9
43	246.0	0.00*	0.1	3.1	68.7
44	251.7	0.00*	0.1	2.2	49.5
45	257.4	0.00*	0.1	1.6	35.3

*LESS THAN OR EQUAL TO .005

distribution. Due to sampling error, this estimate may be somewhat higher or lower than the true 0.001 quantile. Using the second set of tables (Table 4), it is found that with 99% confidence the 0.001 quantile is not below 1.7 years.

V. CONCLUSIONS

In using the tables developed in this review for design of pressure vessels for Air Force applications, it should be kept in mind that the statistical analysis that resulted in Tables 2, 3 and 4, are based solely on strand data. Various arguments have been presented relative to the use of strand data to predict the behavior of filament-wound pressure vessels. Strand data are, however, the only extensive data available on stress rupture of S-Glass/Epoxy composites and should be a consideration in the design of pressure vessels.

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